Spinor Techniques in General Relativity (L24)

Graduate Course

Irena Borzym (12 Lectures) and Peter O'Donnell (12 Lectures)

Spinor structures and techniques are an essential part of modern mathematical physics. This course provides a gentle introduction to spinor methods which are illustrated with reference to a simple 2-spinor formalism in four dimensions. Apart from their role in the description of fermions, spinors also often provide useful geometric insights and consequent algebraic simplifications of some calculations which are cumbersome in terms of spacetime tensors.

The first half of the course will include an introduction to spinors illustrated by 2-spinors. Topics covered will include the conformal group on Minkowski space and a discussion of conformal compactifications, geometry of scri, other simple simple geometric applications of spinor techniques, zero rest mass field equations, Petrov classification, the Plucker embedding and a comparison with Euclidean spacetime. More specific references will be provided during the course and there will be worked examples and handouts provided during the lectures.

The second half of the course will include: Newman-Penrose (NP) spin coefficient formalism, NP field equations, NP quantities under Lorentz transformations, Geroch-Held-Penrose (GHP) formalism, modified GHP formalism, Goldberg-Sachs theorem, Lanczos potential theory, Introduction to twistors. There will be no problem sets.

Pre-requisites

The Part 3 general relativity course is a prerequisite.

No prior knowledge of spinors will be assumed.

Literature

Introductory material.

1. L. P. Hughston and K. P. Tod, *Introduction to General Relativity.* Freeman, 1990.
2. C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation.* Freeman, 1973.

Best Course Reference Text for Lectures 1 to 12.

J.M. Stewart, *Advanced General Relativity.* CUP, 1993.

Best Course Reference Text for Lectures 13 to 24.

P O'Donnell, *Introduction to 2-spinors in general relativity.* World Scientific, 2003.

Reading to complement course material.

1. Penrose and Rindler, *Spinors and Spacetime Volume 1.* Cambridge Monographs on Mathematical Physics, 1987.

2. S. Ward and Raymond O. Wells, *Twistor Geometry and Field theory.* Cambridge Monographs on Mathematical Physics, 1991 .

3. Robert J. Baston, Michael G. Eastwood, *The Penrose Transform.* Clarendon Press, 1989.

4. S. A Huggett and P. Tod, *Introduction to Twistor Theory.* World Scientific, 2003.

5. R.M. Wald, *General Relativity.* World Chicago UP, 1984.

6. S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Spacetime.* CUP, 1973.