Symmetries, Fields and Particles (M24)

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This course introduces the theory of Lie groups and Lie algebras and their applications to high energy physics. The course begins with a brief overview of the role of symmetry in physics. After reviewing basic notions of group theory we define a Lie group as a manifold with a compatible group structure. We give the abstract definition of a Lie algebra and show that every Lie group has an associated Lie algebra corresponding to the tangent space at the identity element. Examples arising from groups of orthogonal and unitary matrices are discussed. The case of $SU(2)$, the group of rotations in three dimensions is studied in detail. We then study the representations of Lie groups and Lie algebras. We discuss reducibility and classify the finite dimensional, irreducible representations of $SU(2)$ and introduce the tensor product of representations. The next part of the course develops the the theory of complex simple Lie algebras. We define the Killing form on a Lie algebra. We introduce the Cartan-Weyl basis and discuss the properties of roots and weights of a Lie algebra. We cover the Cartan classification of simple Lie algebras in detail. We describe the finite dimensional, irreducible representations of simple Lie algebras, illustrating the general theory for the Lie algebra of $SU(3)$. The last part of the course discusses some physical applications. After a general discussion of symmetry in quantum mechanical systems, we review the approximate $SU(3)$ global symmetry of the strong interactions and its consequences for the observed spectrum of hadrons. We introduce gauge symmetry and construct a gauge-invariant Lagrangian for Yang-Mills theory coupled to matter. The course ends with a brief introduction to the Standard Model of particle physics.

Pre-requisites

Linear algebra including direct sums and tensor products of vector spaces. Basic finite group theory, including subgroups and orbits. Special relativity and quantum theory, including orbital angular momentum theory and Pauli spin matrices. Basic ideas about manifolds, including coordinates, dimension, tangent spaces.

Literature

1. J. Fuchs and C. Schweigert, *Lie Algebras and Representations.* Cambridge University Press, 2003.

2. H.F. Jones, *Representations and Physics.* 2nd edition. Taylor and Francis, 1998.

3. H. Georgi, *Lie Algebras in Particle Physics.* Westview Press, 1999.

Additional support

A set of course notes will provided as handouts in the lectures. Printed notes of previous version of the course are also available on the Part III Examples and Lecture Notes webpage. Four examples sheets will be provided and four associated examples classes in moderate-sized groups will be given by graduate students.