

# Algebraic Topology (M24)

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Algebraic topology assigns algebraic invariants (groups and homomorphisms) to topological spaces and continuous maps between them. The most important example of such an invariant is ordinary homology theory, which is part of the basic language of geometry today. This course will cover homology and cohomology, together with applications to the topology of manifolds and vector bundles. The emphasis will be on learning to compute and use these invariants in a variety of examples. A tentative syllabus is as follows:

- *Homology*. Singular homology and cohomology. Eilenberg-Steenrod axioms and cellular homology. The Hurewicz homomorphism.
- *Cohomology and products*. Cohomology and the universal coefficient theorem. The Künneth theorem and cup products.
- *Vector Bundles*. Vector bundles and principal bundles. The long exact sequence of a fibration. The Euler class and the Thom isomorphism.
- *Topology of Manifolds*. Handlebodies. The fundamental class of an oriented manifold. Poincaré duality. The Lefschetz fixed-point theorem.

## Pre-requisite Mathematics

The only required background is basic point-set topology, but those who have not had a previous course in algebraic topology should expect to work hard to keep on top of the material in the first eight lectures, which will be quite rapid. The material in the Michaelmas term Differential Geometry course will be useful as well.

## Literature

1. A. Hatcher, *Algebraic Topology*, CUP (2002).
2. D. Fuchs and A. Fomomenko, *Homotopical Topology*, (2nd ed.) Springer, 2016.
3. J.P. May, *A Concise Course in Algebraic Topology*, University of Chicago Press (1999).
4. R. Bott and L. Tu, *Differential Forms in Algebraic Topology*, Springer (1982).
5. J.W. Vick, *Homology Theory*, Springer (1994).

Hatcher is the standard text, and most closely matches the course syllabus. Vick is a more terse alternative, May more advanced. Bott and Tu and Fuchs and Fomenko offer different perspectives on some of the topics in the course.

## Additional support

Four examples sheets will be provided, each with an associated examples class. There will be a two-hour revision class in the Easter Term.