Introduction to discrete analysis (M16)

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The aim of this course is to explain various key results and techniques in combinatorics, with an emphasis on parts of the subject with an analytic flavour and parts that have connections to other branches of mathematics such as harmonic analysis, analytic number theory and theoretical computer science (though many of these connections will not be made explicit).

The following is an approximate guide to the content of the course. Figures in square brackets represent the rough number of lectures needed for each section.

Discrete Fourier analysis. Roth's theorem, Bogolyubov's method. [2]

Sumsets. Plünnecke's theorem, the Balog-Szemerédi theorem, Freiman-type theorems. [3]

Quasirandomness. Quasirandom graphs, bipartite graphs, and subsets of finite Abelian groups, Szemerédi's regularity lemma, the triangle removal lemma. [4]

The polynomial method. Dvir's theorem, the combinatorial Nullstellensatz, solution of the cap-set problem. [2]

Introduction to higher-order Fourier analysis. Box norms, uniformity norms, related inequalities, Szemerédi's theorem for progressions of length 4 (some parts stated without proof or and some parts sketched). [5]

Prerequisites

There are very few prerequisites. It would be useful to know the basic definitions of graph theory and (discrete) probability. The discrete Fourier analysis will start from first principles, though a prior acquaintance with Fourier analysis will help you put it in context.

This lack of prerequisites does not necessarily make the course easy: experience suggests that the examples sheets, which are important if you want to get the most out of the course, are found quite hard.

Literature

Probably the best source of literature is the internet, where many different accounts of the topics covered in this course can be found. The nearest there is to a standard textbook in the area (which, however, covers far more material than this course will, and in a rather more abstract style) is the following.

> T. C. Tao and V. Vu, *Additive Combinatorics*, Cambridge Studies in Advanced Mathematics **105**, Cambridge University Press.

Additional support

There will be three problem classes, with an examples sheet for each one, and possibly an additional sheet after the course has finished to give further practice in techniques of the course. There will also be a revision session in the summer term.